COMPUTATIONAL SOLUTIONS IN BODY-FITTED COORDINATES OF ELECTRIC FIELDS IN EXTERNALLY SUSTAINED DISCHARGES

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SUMMARY

Computer codes for modeling electric fields within the cavity of externally sustained electric discharge lasers have been developed. These codes use semidirect/marching methods in body-fitted coordinates and are fast enough to make interactive design practical.

Introduction

The objective of the computational effort described herein was to develop computer codes for rapidly and accurately modeling the electric fields within the cavity of pulsed electric lasers such as the VIPER laser (a program at the Air Force Weapons Laboratory). These codes should be fast enough to make the interactive design process practical. The designer should be able to perturb the laser operating parameters and/or the electrode geometry, and quickly obtain new solutions, graphically presented.

In the design of electron beam lasers, it is desirable to have a nearly uniform energy deposition throughout the cavity. This energy deposition is governed by the solution of the non-linear elliptic equation for electric potential ϕ, given by

$$\nabla \cdot \sigma\nabla\phi = 0 \tag{1}$$

where the conductivity σ is a non-linear function of the electric field $E = \nabla\phi$. The solution of this equation for a reasonable grid resolution in two dimensions is a time-consuming effort using conventional methods. Since breakdown depends on the field near the electrode surfaces, accurate representation of the surfaces is essential for realistic modeling.

Solution of the Potential Equation in Cartesian Coordinates by Semidirect Methods

The approach used in the current work is to solve this equation using semidirect non-linear solution techniques previously developed for fluid dynamics applications by the present author. (See, e.g., Refs. 1-3.) The equations are first linearized about some initial guess for the solution. The linear equations so generated are generally variable coefficient, nonseparable, linear elliptic equations. These are solved using marching methods for linear equations which are the only practical way (outside of brute force direct Gaussian elimination) for solving such non-separable elliptic equations directly rather than iteratively. A sequence of linear solutions is then used in a quasi-Picard iteration to solve the nonlinearity. Typically, the solution of the non-linear equations is converged within 10-15 iterations.

The ionization S of the external electron beam gun is modeled empirically, following Ref. 9, by the following equation.

$$S = \exp(-\gamma x)\ \mathrm{atan}\left(\frac{y+a}{x}\right) - \mathrm{atan}\left(\frac{y-a}{x}\right) \tag{2}$$

where $\gamma = E/2 \cdot V$. The electron beam has a voltage V and a width 2a located at x = 0 between y = -a and y = +a. From the same reference, the non-linear conductivity σ is given by

$$\sigma = C \cdot E^{0.45} S^{0.5},\ C = O(1) \tag{3}$$

Initially, the laser cavity was modeled in a cartesian coordinate system with straight electrodes. In this system, we can obtain completely converged non-linear solutions in approximately 5 seconds on a 31x31 grid using a time-shared CDC 6600 computer. This is two orders of magnitude faster than the computer time necessary to solve the problem using a triangular finite element system.

An example of the solution obtained with this method in cartesian coordinates is presented in Figures 1 and 2. Contour plots are presented for the electric field E and the energy deposition rate = σE^2.

Solution of a Body-Fitted Coordinate Transformation by Semidirect Methods

The next step in this research was to obtain solutions of the non-linear potential equation in realistic cavity geometries with non-rectangular cathode and anode shapes. It is sometimes possible to obtain such solutions using a cartesian coordinate system with partial cells near the irregular boundaries. However, it is well-known that such a treatment often leads to numerical instabilities and/or numerical inaccuracies. We have chosen to use the approach of using a numerically generated body-fitted coordinate system, following Thompson, et al. [10,11].

In the body-fitted coordinate system, the physical domain (x,y) is transformed to a regular, rectangular coordinate system in coordinates (ξ,η). In order to insure that no coordinate lines cross in the transformed (ξ,η) plane, the coordinate transformation is generated by the solution of a coupled pair of non-linear elliptic equations for x and y given by

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = 0 \tag{4a}$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = 0 \tag{4b}$$

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14. ABSTRACT

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where

$$\alpha \equiv x_\eta{}^2 + y_\eta{}^2 \quad (4c)$$

$$\beta \equiv x_\xi x_\eta + y_\xi y_\eta \quad (4d)$$

$$\gamma \equiv x_\xi{}^2 + y_\xi{}^2 \quad (4e)$$

These equations are solved for $x(\xi,\eta)$ and $y(\xi,\eta)$, with the boundary values on x and y being specified by the boundary shape and the desired mesh density at the boundaries. Second-order accurate finite difference equations are used throughout.

Note that all calculations, including the calculation for the generation of the coordinates, are performed in the (ξ,η) plane rather than the physical (x,y) plane. Thompson, et al. [10,11] use a point SOR iteraton scheme to solve the coordinate system transformation, but this proves to be inefficient for high mesh resolution. We again use a semidirect method to solve the coordinate system transformation. This system is well-suited to the semidirect method because the two non-linear equations for $x(\xi,\eta)$ and $y(\xi,\eta)$ couple only at interior points. (This contrasts to the boundary coupling of the Navier-Stokes equations, which slows convergence. See Refs. 1-3.)

The speed of convergence of these non-linear equations for the coordinate systems depends on the accuracy of the initial guess. For a mild problem in which the electrode shapes are a slight distortion from rectangular, an adequate initial guess is obtained by simple linear interpolation between boundary values of x and y in the (ξ,η) plane. In these cases, the coordinate system transformation is solved typically in less than 10 non-linear quasi-Picard iterations. An example of the coordinates so generated is shown in Figure 3. For more severe problems, adequate initial guesses are difficult to achieve. (For example, even for a simple U-shaped cavity, linear interpolation gives a folded coordinate system with a negative Jacobian.) For these cases, it will be necessary to build up to the final geometry by way of a sequence of intermediate geometries, using non-linear continuation methods. This aspect of the work is still under development.

Solution of the Potential Equation in Body-Fitted Coordinates

The transformed non-linear potential equation is now to be solved in the (ξ,η) plane. However, the form of the equation changes drastically, because the coordinate transformation used is non-conformal. In particular, mixed or cross derivatives of the form $\partial^2\phi/\partial\xi\partial\eta$ are generated where none existed in the physical plane. This introduces no inaccuracy in the solution, but does require that the solution method used to solve the equations be able to treat a general 9-point operator. Again, the marching methods for elliptic equations are capable of handling this generality, although at some penalty in computer time.

For simple geometries like that shown in Figure 3, in either planar or cylindrical geometries, the non-linear solution on a 25x25 grid is obtained in 5 to 15 seconds on a CDC 6600.

Interactive Graphics

An important goal of the present work is to enable the researcher to interactively design electrode shapes and laser operating parameters so as to achieve a near uniform energy deposition the laser cavity. This will be possible because of the rapid solution time of the semidirect methods, coupled within interactive software currently under development.

The researcher in laser physics will use a Tektronix 4662 plotter to digitize the electrode and cavity shapes and interactively generate body-fitted coordinate systems, then interactively generate solutions, perturb the parameters of the problem, and finally obtain acceptable solutions and contour plots of the solution functionals.

Future Work

In the immediate future, we intend to develop the three areas described below.

(1) Coordinate system control via the non-homogeneous terms[10,11] in (4).

(2) Coordinate solutions for severe geometries via non-linear continuation.

(3) Interactive design capability via digitizing and graphics displays.

In the longer range, we intend to pursue the following areas.

(4) Improved solution for non-linear conductivity, possibly using a Monte Carlo solution.

(5) Improved far-field boundary conditions, possibly using asymptotic solutions.

(6) Three-dimensional solutions.

(7) Fourth-order accurate solutions.

(8) One-dimensional sheath solution of the charge separation equations, possibly with Townsend coefficients and diffusion, coupled to the multidimensional electric field codes.

(9) Automation of the design process to achieve a near uniform energy deposition.

(10) Time dependent solutions.

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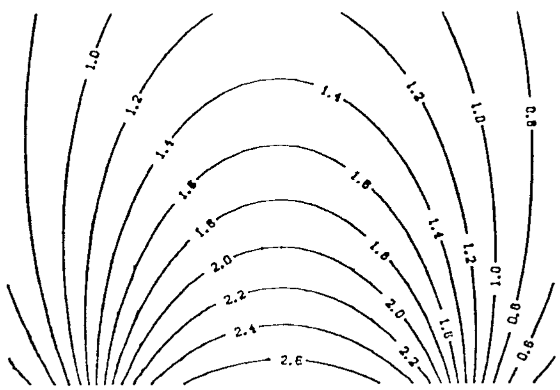

Figure 1. Example of electric field solution in rectangular coordinates.

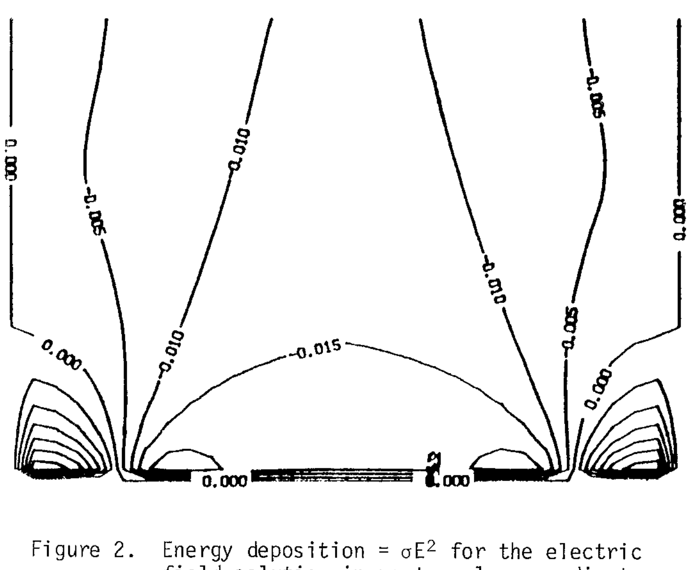

Figure 2. Energy deposition = σE^2 for the electric field solution in rectangular coordinates.

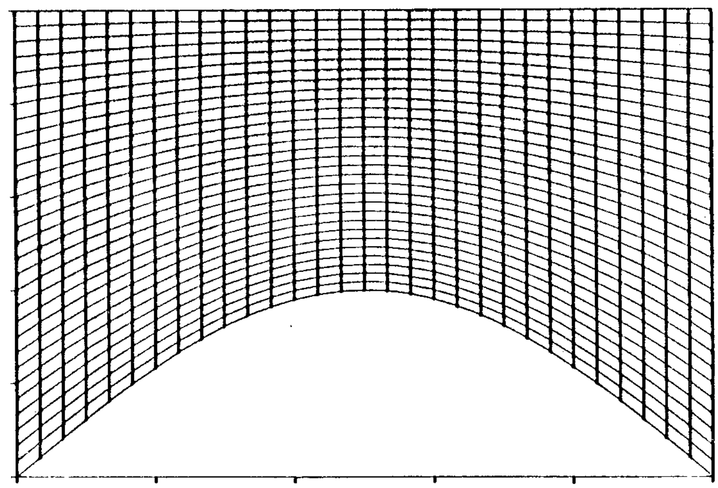

Figure 3. Example of solution of body-fitted coordinates for a mild geometry.

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